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Use of the Method of Guidance by a Required Velocity in Control of Spacecraft Attitude

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ARTICLE INFO

Article history

Received: 17 September 2021

Accepted: 28 October 2021

Published Online: 3 November 2021

Keywords:

Method of guidance by a required velocity

Iterative control

Free trajectory method

Terminal control

Quaternion

Spacecraft attitude

Prognostic model

ABSTRACT

We apply the method of guidance by a required velocity for solving the optimal control problem over spacecraft's reorientation from known initial attitude into a required final attitude. We suppose that attitude control is carried out by impulse jet engines. For optimization of fuel consumption, the controlling moments are calculated and formed according to the method of free trajectories together with principle of iterative control using the quaternions for generating commands to actuators. Optimal solution corresponds to the principle "acceleration - free rotation - separate corrections - free rotation - braking". Rotation along a hitting trajectory is supported by insignificant correction of the uncontrolled motion at discrete instants between segments of acceleration and braking. Various strategies of forming the correction impulses during stage of free motion are suggested. Improving accuracy of achievement of spacecraft's final position is reached by terminal control using information about current attitude and angular velocity measurements for determining an instant of beginning of braking (condition for start of braking based on actual motion parameters is formulated in analytical form). The described method is universal and invariant relative to moments of inertia. Developed laws of attitude control concern the algorithms with prognostic model, the synthesized control modes are invariant with respect to both external perturbations and parametric errors. Results of mathematical modeling are presented that demonstrate practical feasibility and high efficiency of designed algorithms.

1. Introduction

Control problem of turning the spacecraft into given angular position from arbitrary initial attitude in a finite time t_f with minimization of propellant consumption and given accuracy of reorientation was solved. Spacecraft motion around the center of mass is described by quaternion of attitude ^[1]. Designing the optimal rotation program is based on quaternion models, method of free trajectories, and method of iterative guidance as particular case of the method of guidance by a required velocity ^[2].

Now, spacecrafts are used in many areas of scientific occupations and industry. In particular, astrophysical researches and other scientific discoveries would be impossible without modern spacecrafts ^[3-5]. Success of mission and duration of performance in a working point of orbit (orbital position) are provided by successful control of motion, by an efficiency of attitude control (an improved system of spacecraft attitude is especially important for the spacecrafts with instruments and devices for astronomy measurements and for satellites of Earth supervision).

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Numerous papers study control problems of rigid bodies in various statements [6-29]. For spherically symmetric body, time-optimal spatial rotation is known [1]. Diverse methods are used for constructing control program of spacecraft orientation, in particular, algorithm of fuzzy logic [7] or concept of inverse problem of dynamics [8,9]. Finding the optimal solution of spacecraft's motion control is known also [10-24]. Time-optimal maneuvers are more popular [11-20]. Some solutions are obtained for axially symmetric spacecraft [19-22]. Terminal control for orbital orientation of a spacecraft was considered also [25]; controlling the spacecrafts with control moment gyroscopes has features [26-28]. Optimization of spacecraft attitude with minimum fuel consumption is a difficult issue in mathematical aspect (and difficult engineering problem, also). This paper describes optimal program of spatial turn of arbitrary spacecraft realizing the mode of guidance by a required velocity and method of free trajectories. We give numerical estimates of fuel expenditure for realization of a turn taking into account disturbances acting upon the spacecraft (in particular, gravitational and aerodynamic torques). Issues of economical control of spacecraft motion are still relevant and topical today, so the solved problem of a turn is practically important.

2. Angular Motion's Equations and Statement of Control Problem

We consider the case when parameters of a turn (for example, components of turn quaternion) are known in advance, even before the beginning of maneuver; any initial angular differences are possible (from a few degrees up to 180 degrees), angular orientation of right-hand coordinate system *OXYZ* related with a spacecraft (as well as its initial and final positions) being determined relative to a chosen reference basis. It is assumed that the reference system coincides with inertial coordinate system (inertial basis *I*), as the most popular case. Spacecraft rotation satisfies dynamical equations [1,6]:

$$\begin{aligned} J_1 \dot{\omega}_1 + (J_3 - J_2) \omega_2 \omega_3 &= M_1, \\ J_2 \dot{\omega}_2 + (J_1 - J_3) \omega_1 \omega_3 &= M_2, \\ J_3 \dot{\omega}_3 + (J_2 - J_1) \omega_1 \omega_2 &= M_3 \end{aligned} \quad (1)$$

where J_i are central principal moments of inertia of spacecraft, M_i are projections of torque **M** onto principal axes of spacecraft's inertia ellipsoid, ω_i are projections of spacecraft's absolute angular velocity vector **ω** onto axes of body basis **E** formed by the principal central axes of spacecraft's inertia ellipsoid ($i = \overline{1, 3}$). Spacecraft attitude is described by known equation [1].

$$2\dot{\Lambda} = \Lambda \circ \omega \quad (2)$$

where ω is vector of absolute angular velocity of spacecraft; Λ is quaternion of orientation with respect to basis **I** (we assume $\|\Lambda(0)\| = 1$). Equation (2) has the boundary conditions $\Lambda(0) = \Lambda_{in}$ and $\Lambda(T) = \Lambda_f$, where T is time of termination of a turn (Λ_{in} and Λ_f have any a priori given values which satisfy the condition $\|\Lambda_{in}\| = \|\Lambda_f\| = 1$ (because quaternion Λ is normalized). We assume that initial and final angular velocities are equal to zero: $\omega(0) = \omega(T) = 0$. If spacecraft's actuators are jet engines which control rotations about three axes of a spacecraft, general form of index for fuel expenditure is

$$G = \int_0^T \left(\frac{|M_1|}{I_1} + \frac{|M_2|}{I_2} + \frac{|M_3|}{I_3} \right) dt \quad (3)$$

where I_1, I_2 , and $I_3 > 0$ are the arms of thrust action of jet engines (for attitude control) in channels *x, y, z*. Optimization problem of spacecraft's spatial reorientation is a finding a control rotating the spacecraft from position Λ_{in} into position Λ_f and minimizing the index (3) under constraint [1,11]:

$$M_1^2 + M_2^2 + M_3^2 \leq m_0^2 \quad (4)$$

and requirement $T \leq T_{des}$, where m_0 is maximal possible magnitude of control torque **M** which actuators can give (m_0 characterizes a power of spacecraft's actuators); T_{des} is a desired duration of turn.

3. Solving the Formulated Problem of Controlled Turn

We assume that spacecraft is solid (it is rigid body). Optimal control of three-dimensional turn must rotate spacecraft from attitude to the Λ_{in} e required attitude Λ_f according to the Equations (1), (2) with minimal functional (3). When finding optimal law of rotation (in sense of minimum (3)), we assume that angular velocity $\omega(t)$ is a piecewise continuous time function. It is known [23], spacecraft's rotation optimal in fuel consumption includes two phases with maximal control torque (segment of acceleration and segment of braking), and free motion phase when control torque is absent. This type of controlling reorientation of a spacecraft is called two-pulse control (first pulse for imparting the required angular momentum **L** to spacecraft's body, and second pulse suppress angular velocity). Taking into account that disturbances act slightly (disturbance torque much less control torque), we suggest that free rotation is optimal for arbitrary spacecraft in sense of consumption G (since control moment M_c is absent). Free rotation is described by known system of equations

$$\dot{\omega}_1 = k_1 \omega_2 \omega_3, \quad \dot{\omega}_2 = k_2 \omega_1 \omega_3, \quad \dot{\omega}_3 = k_3 \omega_1 \omega_2 \quad (5)$$

where $k_1 = (J_2 - J_3)/J_1$, $k_2 = (J_3 - J_1)/J_2$, and $k_3 = (J_1 - J_2)/J_3$ are constant coefficients.

Idea of the proposed principle of control consists in determining such angular momentum $\mathbf{L}^*$ under which spacecraft transfers in given attitude Λ_f under free motion. In this case, fuel is expended for spacecraft's acceleration and braking (for increasing the angular velocity and damping of rotation). Reorientation itself is fulfilled without control torque ($\mathbf{M}_c = 0$), and therefore, regime of fuel economy is made practically along entire trajectory of motion. For this approach, basic turn is executed with zero fuel expenditure, and mode of a controlled turn is optimum. Possibility of designing the optimal solution in class of two-impulse control is justified by fact that each segment of attitude trajectory is optimal for chosen criterion. Expense is $G = G_{ac} + G_{fm} + G_{br}$, where G_{ac} is fuel expenditures during acceleration of a spacecraft, G_{br} is expenditures during a braking of a spacecraft, and G_{fm} is fuel expenditures within phase of spacecraft's free rotation (between acceleration and braking).

For slow maneuver, important characteristic is an integral

$$S = \int_0^T |\mathbf{L}(t)| dt \quad (6)$$

The value S is determined only by rotation conditions Λ_{in} , Λ_f , and spacecraft's principal central moments of inertia J_1, J_2, J_3 . The calculated value S is $S = K_c t_{ex}$, where K_c is arbitrary magnitude of angular momentum ($K_c > 0$); t_{ex} is the expected time of reorientation from position Λ_{in} into position Λ_f , i.e. time when equality $\Lambda = \Lambda_f$ holds for solution $\Lambda(t)$ of system of Equations (2), (5) with initial conditions $\omega(0) = K_c \omega^* / J_{sc} \omega^*$, $\Lambda(0) = \Lambda_{in}$ (the corresponding theorem can be proven [24]), where ω^* is solution of the boundary problem $\Lambda(0) = \Lambda_{in}$, $\Lambda(t_f) = \Lambda_f$, taking into account the Equations (2), (5); t_f is time of arrival to position Λ_f obtained by simulation of motion according to the Equations (2), (5).

The boundary value problem $\Lambda(t_j) = \Lambda_{in}$, $\Lambda(t_f) = \Lambda_f$, for the system of Equations (2), (5), has analytical solution (in elementary functions) only for dynamically symmetric and dynamically spherical bodies. For spherically symmetric spacecraft (when $J_1 = J_2 = J_3$), solution $\mathbf{p}(t)$, $\boldsymbol{\omega}(t)$ have elementary form: $\mathbf{p}(t) = \text{const}$ and $\boldsymbol{\omega}(t) = \text{const}$, or in detail

$$p_i = v_i / \sqrt{v_1^2 + v_2^2 + v_3^2}, \text{ and } \omega_i = \frac{2v_i \arccos v_0}{T \sqrt{v_1^2 + v_2^2 + v_3^2}}$$

where v_0, v_1, v_2, v_3 are components of turn quaternion

$$\Lambda_t = \tilde{\Lambda}_{in} \circ \Lambda_f \quad [1]. \text{ Characteristic (6) is equal to } S = 2J_1 \arccos v_0.$$

For a dynamically symmetric body (when, for example, $J_2 = J_3$), the optimal control problem is solved completely. We write optimal solution $\boldsymbol{\omega}(t)$ in the following form:

$$\omega_1 = \dot{\alpha} + \dot{\beta} \cos \vartheta, \omega_2 = \dot{\beta} \sin \vartheta \sin(\dot{\alpha} t + \sigma), \omega_3 = \dot{\beta} \sin \vartheta \cos(\dot{\alpha} t + \sigma)$$

where $\sigma = \arctg(p_{20} / p_{30})$; ϑ is the angle between the spacecraft's longitudinal axis and the vector $\mathbf{p}$ ($0 \leq \vartheta \leq \pi$); $\dot{\alpha}$ is the angular velocity of its own rotation (around the longitudinal axis); and $\dot{\beta}$ is the angular velocity of the precession (around the vector $\mathbf{p}$). Characteristic (6) is equal to

$$S = \sqrt{J_1^2 (\alpha + \beta \cos \vartheta)^2 + J_2^2 \beta^2 \sin^2 \vartheta}$$

where α is angle of turn about longitudinal axis; β is angle of turn about vector $\mathbf{p}$ (note, $p_{10} = \cos \vartheta$). Optimal values of parameters p_0 , α , β , and ϑ are determined by the boundary angular positions Λ_{in} and Λ_f through the system of equations [23]

$$\begin{aligned} \cos \frac{\beta}{2} \cos \frac{\alpha}{2} - p_{10} \sin \frac{\beta}{2} \sin \frac{\alpha}{2} &= v_0, \\ \cos \frac{\beta}{2} \sin \frac{\alpha}{2} + p_{10} \sin \frac{\beta}{2} \cos \frac{\alpha}{2} &= v_1, \\ p_{20} \sin \frac{\beta}{2} \cos \frac{\alpha}{2} + p_{30} \sin \frac{\beta}{2} \sin \frac{\alpha}{2} &= v_2, \\ -p_{20} \sin \frac{\beta}{2} \sin \frac{\alpha}{2} + p_{30} \sin \frac{\beta}{2} \cos \frac{\alpha}{2} &= v_3 \end{aligned} \quad (7)$$

concurrently with condition $J_1(\alpha + \beta \cos \vartheta)^2 + J_2 \beta^2 \sin^2 \vartheta \rightarrow \min$ ($|\alpha| \leq \pi$, $0 \leq \beta \leq \pi$), where J_1 is the moment of inertia about longitudinal axis of a spacecraft; J_2 is the moment of inertia about transverse axis of a spacecraft. Optimal values of vector $\mathbf{p}_0$ and values α , β , and ϑ , which satisfy the given attitudes Λ_{in} and Λ_f in initial and final instants, can be determined with use of known device [30].

It is essential that many known methods are unsuitable for situations when initial angle of turn between attitudes Λ_{in} and Λ_f is large. Many researchers use method of combining synthesis which use predictive model. But such algorithms give final result and control program that completely depends from the assumed form of predictive model (the chosen model of motion forecast completely determines type of controlled rotation during maneuver). Any author has insuperable mathematical difficulties if takes predictive model even little close to reality. Below we consider one method of optimal reorientation which uses the method of guidance by a required velocity and method of free trajectories.

4. Application of the Method of Guidance by a Required Velocity for Controlled Maneuver

Method's essence consists in periodical correction of spacecraft's attitude trajectory at specified instants of time. Control is reduced to correction for which onboard computer

determines angular velocity that is necessary for reaching the given attitude Λ_f under free rotation, calculation of the desired angular momentum, corresponding to the calculated angular rate, and a transferring a correction impulse $\Delta \mathbf{L}$ to spacecraft's body having angular momentum $\mathbf{L}$, if the latter is substantially different from the desired value. Every time, guidance is carried out from current attitude $\Lambda(t)$ to the given position Λ_f . Angular rates that are required for next site of attitude trajectory are calculated by condition of minimum fuel expenditure for further control of spacecraft rotation. The used predictive model has specific feature, the form of this model gives forecast of free rotation in class of spacecraft motion along conical trajectories where direction of angular momentum of dynamically symmetric body is constant in inertial coordinate system. Such approach allows us to solve problem of constructing the optimal control of arbitrary spacecraft turn using the iterations method. Free rotation of a spacecraft is a combination of two motions: precession of longitudinal axis OX about angular momentum vector $\mathbf{L}$ and spacecraft's rotation itself about longitudinal axis OX .

For axial-symmetric body ($J_2=J_3$) the rates of precession $\dot{\beta}$ and proper rotation $\dot{\alpha}$ are constant and connected between themselves by the dependence: $\dot{\alpha} = \dot{\beta} (J/J_1 - 1) \cos \vartheta$, where J is moment of inertia with respect to transverse axis, J_1 is moment of inertia with respect to longitudinal axis, and ϑ is angle of nutation (angle between longitudinal axis OX and angular momentum $\mathbf{L}$). The desired vector $\mathbf{L}^*$ runs in the plane which is perpendicular to plane $X_s O X_f$ and is deviated from axis OX on angle ϑ that guarantees spacecraft's rotation simultaneously through angles α and β in time T_{des} (we note that X_s and X_f are the directions of spacecraft's longitudinal axis before and after reorientation).

Situations when boundary rates $\omega(0) = \omega(T) = 0$ (such conditions of spacecraft turn are most typical) are of practical importance. Of course, at times $t=0$ and $t=T$ angular rate for nominal rotation program are not zero. Consequently, transfer phases are necessary: acceleration of rotation as transition from state of rest (when $\omega = 0$) to regime of rotation with angular momentum of maximum magnitude L_m , and braking, i.e. reduction of spacecraft's angular rate to zero (value L_m is specified by turn duration T). Between acceleration of rotation and braking, spacecraft carries out free motion.

We find prediction of free rotation in form of regular precession of dynamically symmetric body. Parameters of predictive model are computed using the condition of maximal approximation of the predicted motion to real rotation of a spacecraft. Let us study system of equations that reflects motion within uncontrolled phase ($\mathbf{M}_e = 0$). For many spacecrafts, $J_2 \approx J_3$, but $J_2 \neq J_3$. Further on, for definiteness we suppose $J_2 > J_3$, and J_3 much more than J_1 and $|J_2 - J_3|$ much

less than J_1 . Then the moment $(J_3 - J_2)\omega_2\omega_3$ is insignificant, and we assume it as perturbation (we neglect its influence on prediction). For complete integrability of equations of rotation (including kinematic equations for spacecraft attitude) we use assumption about dynamical symmetry of a spacecraft (for predicting only). Moment of inertia J around transversal axis must satisfy the relationship $J_3 < J < J_2$. For decreasing the errors of model, choice of concrete value J must preserve invariable characteristic equation of the system. Therefore, condition for finding the value J consists in following:

$$\left(\frac{J - J_1}{J} \right)^2 = \frac{(J_2 - J_1)(J_3 - J_1)}{J_2 J_3}$$

since $\omega_1 = \text{const} = \omega_{10}$ in simplified system (i.e. system (1) without the moment $(J_3 - J_2)\omega_2\omega_3$), and cyclic frequency is $f = \omega_{10} \sqrt{(J_3 - J_1)(J_2 - J_1)/(J_2 J_3)}$ (we know that dynamically symmetric body has cyclic frequency $\omega_{10}(J - J_1)/J$ because $\omega_1 = \text{const}$).

Dynamics of real spacecraft during free motion is described by the following system:

$$\begin{aligned} J_1 \dot{\omega}_1 &= (J_2 - J_3)\omega_2\omega_3 + M_{xp} \\ J \dot{\omega}_2 + (J_1 - J)\omega_1\omega_3 &= M_{yp} J / J_2 + \\ &+ (J(J_3 - J_2 - J_1) / J_2 + J_1)\omega_1\omega_3 \\ J \dot{\omega}_3 + (J - J_1)\omega_1\omega_2 &= M_{zp} J / J_3 + \\ &+ (J(J_3 - J_2 + J_1) / J_2 - J_1)\omega_1\omega_2 \end{aligned} \quad (8)$$

where M_{xp} , M_{yp} , M_{zp} are moments of perturbations, and $J = \frac{J_2 J_3}{J_2 + J_3 - J_1} (\sqrt{(1 - J_1/J_2)(1 - J_1/J_3)} + 1)$.

Torques in right-hand parts of Equations (8) are small (they can be assumed as perturbations), and they are neglected in predictive model. Then, we can write predictive model as the following system: $\omega_1 = \omega_{10} = \text{const}$,

$$J \dot{\omega}_2 + (J_1 - J)\omega_1\omega_3 = 0, \quad J \dot{\omega}_3 + (J - J_1)\omega_1\omega_2 = 0 \quad (9)$$

Solving the boundary value problem $\Lambda(t_j) = \Lambda_j$, $\Lambda(t_f) = \Lambda_f$ with (2), (9) taken into account, we will find expressions for calculating the required angular rates ω_{10} , ω_{20} , and ω_{30} (at the beginning of segment of the uncontrolled motion). We remind that j is number of correction, t_j is instant of beginning the correction; first segment of free motion starts with initial angular velocities which satisfy the boundary value problem $\Lambda(0) = \Lambda_{in}$, $\Lambda(t_f) = \Lambda_f$ for dynamic system (2), (9).

Taking into account that distinction between real and the predicted rotation is insignificant, we apply method of iterative

guidance in order to synthesise control program, for impulses of jet engines, during reorientation. In accordance with this principle, entire trajectory of attitude is partitioned in a number of sites within which there is no control (impulses of jet engines are absent). Transition from one site to another site is executed by impulses of correction. There is only one requirement to the sites of uncontrolled rotation: they must pass through positions $\Lambda(t)$ and Λ_f . At instant of correction impulse, the calculated angular velocity (the programmed value) is determined

$$\omega_{i0} = \omega_{i\text{nom}} = L_m p_{i0} / J_i, \text{ and} \\ L_m = m_0 T_{\text{des}} \left(1 - \sqrt{1 - 4S / (m_0 T_{\text{des}}^2)} \right) / 2$$

where p_{i0} are computed by the system (7) in which v_0, v_1, v_2, v_3 are components of quaternion of discrepancy $\Lambda_d = \tilde{\Lambda} \circ \Lambda_f$ at beginning of correction impulse. If correction impulses are carried out continuously then p_{i0} almost not change practically (because correction moments act constantly in this case). If corrections are made periodically and very often, then p_{i0} vary very slightly (insignificantly) but it require the increased expense of fuel also. We offer to correct motion at discrete separate instants of time t_j for decrease of fuel consumption. For example, we can do corrections according to the following law: correction impulse is made at instant when condition $\psi_f = k\psi_0$ is satisfied, and

$$\psi_0 = 2\arccos(\text{sqal}(\tilde{\Lambda}_0 \circ \Lambda)) ; \quad \psi_f = 2\arccos(\text{sqal}(\tilde{\Lambda}_f \circ \Lambda)) , \\ k = \text{const}$$

where t_j is instant of start of motion correction (j is number of correction), ψ_0 is angle of turn from attitude of last previous correction impulse to the current position, and ψ_f is angle of turn from current attitude to final position Λ_f . After each correction $\Lambda_0 = \Lambda(t_n)$, where t_n is instant of acting the correction impulse. For first correction $\Lambda_0 = \Lambda_{\text{in}}$ (before the beginning of a turn $\Lambda_0 = \Lambda_{\text{in}}$).

It is expedient to select value of coefficient k close to unity. When k increases ($k > 1$), size of the uncontrolled sites increases also, perturbations are accumulated, that leads to increasing fuel expenditure. When k decreases ($k < 1$), corrections are made so frequently that control is almost continuous. In this situation, necessary direction of angular momentum is endlessly recomputed (its magnitude remains constant). By virtue of smallness of sites of rotation this direction is also almost constant in inertial coordinate system. This senseless computing expenditure is totally unjustified because it does not reduce fuel consumption in comparison with control when spacecraft rotate along conical trajectory (in form of regular precession with constant angle of nutation).

More best version of strategy for correction of spacecraft rotation is variant when correction impulse is made at half of a hitting trajectory (a predicted motion), i.e. when angle between current position and position preset at ending a controlling impulse (acceleration or correction) is equal to the angle between current position and the required final position. Condition for start of correction is

$$\text{sqal}(\tilde{\Lambda}_0 \circ \Lambda) = \text{sqal}(\tilde{\Lambda}_f \circ \Lambda) .$$

If the controlling moment $\mathbf{M}$ is limited, then a boost of spacecraft angular momentum to the required level $|\mathbf{L}| = L_m$ at beginning of a turn and damping of available angular momentum to zero at end of reorientation maneuver occupy some finite time (distinct from zero). In general case, conditions of turn Λ_{in} and Λ_f may be such that one cannot neglect transition segments (acceleration and braking). Quite often the vector $\mathbf{M}$ obey condition (4). Since initial and final angular velocities are equal to zero and magnitude of control moment is constant $|\mathbf{M}| = \text{const} = m_0$, duration of stages of acceleration and braking is identical. Optimal solution $\omega(t)$ during segment of nominal motion (between acceleration and braking) possesses property $|\mathbf{L}| \approx \text{const}$ (inconstancy of modulus of angular momentum can be due to a presence of disturbing moments and inequality of the moments of inertia $J_2 = J_3$).

The laws of fastest imparting and reduction of angular velocity under constraint (4) are known [11]. At segment of acceleration, optimal control has following form [11]:

$$\mathbf{M} = m_0 J_{SC} \omega / |J_{SC} \omega| \quad (10)$$

where $J_{SC} = \text{diag}(J_1, J_2, J_3)$ is spacecraft's inertia tensor. If differentiate by time last equation, taking into account the Equation (1), then we will obtain the following equations

$$\dot{M}_1 = \omega_3 M_2 - \omega_2 M_3, \quad \dot{M}_2 = \omega_1 M_3 - \omega_3 M_1, \\ \dot{M}_3 = \omega_2 M_1 - \omega_1 M_2$$

which show that $\mathbf{M}$ is constant vector relative to inertial basis $\mathbf{I}$, and $|\mathbf{M}| = \text{const} = m_0$. At optimal motion, angular momentum of a spacecraft does not change direction in inertial coordinate system. Magnitude of angular momentum varies according to the law $|\mathbf{L}| = m_0 t$. At segment of braking, optimal control is

$$\mathbf{M} = -m_0 J_{SC} \omega / |J_{SC} \omega| \quad (11)$$

(the controlling moment $\mathbf{M}$ makes with angular momentum an angle of 180 degree) [11]. Angular momentum varies according to the law $|\mathbf{L}| = L_{oc} - m_0(t - t_{br})$, where $L_{oc} = |J_{SC} \omega(t_{br})|$; t_{br} is time of beginning of damping. For both acceleration and braking, optimal control (as fast response) is control under

which the controlling moment is parallel to angular momentum at any moment of time.

The proposed algorithm performs the control of spacecraft rotation according to method of free trajectories. It presumes correction of spacecraft's rotation at certain discrete instant of time. Entire attitude trajectory consists of alternating the controlled phases and uncontrolled phases, and it includes phases of acceleration and braking, phases of free rotation (when $\mathbf{M}_c = 0$) and short-time phases of correcting the attitude trajectory. Task of control is to provide such start conditions for uncontrolled phases that the predicted trajectory of rotation must pass through final attitude Λ_f . For synthesis of control impulses, quaternion of turn $\Lambda_t^{(n)} = \tilde{\Lambda}(t_n) \circ \Lambda_f$, at beginning of each non-controlled site t_n , is calculated. Using it, initial rates $\omega_{10}, \omega_{20}, \omega_{30}$ are computed for next site of uncontrolled rotation. Usually, from one to three or five correcting impulses (it depends on turn angle) are sufficient for reorientation. Optimization consists in determining the time of rotation's acceleration and damping of rotation. Control torque on segment of acceleration (braking) is specified by conditions: (10) for acceleration segment, and (11) on the braking segment. Control moment remains immobile vector in inertial space during both segments. Duration of acceleration (braking) τ can be determined as $\Delta t_{ac} \approx \Delta t_{br} = \tau =$

$$T_{des} \left(1 - \sqrt{1 - 4S / (m_0 T_{des}^2)} \right) / 2, \text{ and time of free motion is } \Delta t_{free} = T_{des} \sqrt{1 - 4S / (m_0 T_{des}^2)} \text{ (it is assumed that } 4S < m_0 T_{des}^2 \text{). Let us explain it.}$$

For free rotation, integral of modulus of spacecraft's angular momentum S does not depend from time of turn T [24]. If durations of transition periods Δt_{ac} and Δt_{br} (acceleration and braking) are small, and the sum $\Delta t_{ac} + \Delta t_{br}$ much less than T_{des} , then integral of modulus of angular momentum during rotation time T barely changes and remains close to S , and the change of modulus of angular momentum during acceleration and braking can be considered linear. Then we have the equality $(T_{des} - (\Delta t_{ac} + \Delta t_{br}) / 2) L_m = S$, where L_m is modulus of angular momentum at phase of nominal rotation (when $|\mathbf{L}| = \text{const}$); Δt_{ac} and Δt_{br} are durations of acceleration and extinction of angular momentum. We have $\Delta t_{ac} + \Delta t_{br} \geq 2L_m / m_0$, since $\tau = L_m / m_0$ is minimal possible acceleration (braking) time with restriction $|\mathbf{M}| \leq m_0$. Hence $L_m \geq S / (T_{des} - \tau)$ and $T_{des} \approx S / L_m + L_m / m_0$ (since times of acceleration Δt_{ac} and braking Δt_{br} are equal).

As a result, control for spacecraft's spatial reorientation consists in following operations:

(1) The computing the turn quaternion $\Lambda_t = \tilde{\Lambda}_{in} \circ \Lambda_f$, and the determining the required angular velocities ω_{10}, ω_{20} , and ω_{30} for next uncontrolled phase.

(2) Acceleration of rotation to the calculated angular momentum $\mathbf{L}^*$ under control

$$\mathbf{M}_c = m_0(\mathbf{L}^* - \mathbf{L}) / |\mathbf{L}^* - \mathbf{L}|,$$

where $\mathbf{L}^* = \tilde{\Lambda} \circ \Lambda_{in} \circ \mathbf{L}_{pr} \circ \tilde{\Lambda}_{in} \circ \Lambda$, and $\mathbf{L}_{pr}$ is preset vector of angular momentum with components $J_i \omega_{i0}$;

(3) Free rotation ($\mathbf{M}_c = 0$) until instant t_n , when $\text{sqal}(\tilde{\Lambda}_0 \circ \Lambda(t_n)) = \text{sqal}(\Lambda(t_n) \circ \tilde{\Lambda}_f)$.

(4) At instant t_n one should calculate new quaternion of turn and compute initial rates ω_{in} for new site of attitude trajectory (new hitting trajectory). Then the controlling impulse $\Delta \mathbf{L}$: $\Delta L_i = J_i(\omega_{in} - \omega_i)$ is calculated. Control torques are computed $M_i = \Delta L_i / \Delta t$, where Δt is calculated from constraint (4) (Δt is minimum possible value but such that constraint (4) is valid).

Then one should set $t_0 = t_n$ and repeat items (3) and (4) until instant for which $2\psi_f > |\omega| |\mathbf{L}| / m_0$.

(5) Damping of spacecraft's rotation using control torque (11) for which $\mathbf{M}_c \cdot \mathbf{L} < 0$, $\mathbf{M}_c = \tilde{\Lambda} \circ \mathbf{M}_{br} \circ \Lambda$, where $\mathbf{M}_{br} = \Lambda_{br} \circ \mathbf{L}_{br} \circ \tilde{\Lambda}_{br}$, and $\mathbf{L}_{br}$, Λ_{br} are angular momentum and quaternion of spacecraft attitude at instant of start of braking (i.e., control torque is directed exactly against angular momentum, and direction of controlling torque is constant in inertial basis).

The proposed algorithm of spacecraft's attitude control was patented earlier [29]. Angular velocities ω_{in} required for next site of uncontrolled trajectory are determined using the condition of minimum consumption for control of maneuver terminating. Evidently, in neighborhood of the programmed angular rate ω^* we can assume that $\omega_{in} = \chi \omega_{in}^*$, i.e. direction of angular rate is immobile (it is known after calculating the vector ω^*), and modulus of angular velocity vector must be optimized: $|\omega| \rightarrow \text{var}$. Therefore, consumption of fuel for maneuver terminating is function of single parameter χ . Spacecraft's rotation begins to be damped since the instant when equality $2\psi_f = |\omega| |\mathbf{L}| / m_0$ becomes satisfied.

Optimal control problem of three-dimensional reorientation was solved applying algorithm of joint synthesis based on use of predictive model. Constructed control law is quasi-optimal and invariant, and it does not require exact knowledge of rotation model parameters. Efficiency in sense of energy-saving control is reached by the mode when control torque is absent during main part of maneuver ($\mathbf{M}_c = 0$), and high precision is ensured by the constructing the feedback with use of data about spacecraft's attitude and angular velocity when control torques are formed and generated.

5. Application of the Method of Guidance by a Required Velocity with Prediction

For improving an accuracy of reorientation and for decrease of fuel consumption, we can use information about calculated prognostic position Λ^* at instant of satisfaction of condition $\text{sqa}(\tilde{\Lambda}_{\text{in}} \circ \Lambda) = \text{sqa}(\tilde{\Lambda} \circ \Lambda_f)$, which is obtained for accurately predicted trajectory passed through current position Λ and the required final position Λ_f (i.e. $\text{sqa}(\tilde{\Lambda}_{\text{in}} \circ \Lambda^*) =$

$\text{sqa}(\Lambda^* \circ \tilde{\Lambda}_f)$). Taking into account that real spacecraft motion only slightly differs from the predicted one, let us employ method of iterative guidance in order to form the control moments in process of a turn. Its essence consists of regular correction of spacecraft motion trajectory at fixed moments of time. Correction consists of determining the angular momentum $\mathbf{L}^*$, which is necessary for attaining the final position Λ_f , and of imparting the correcting impulse $\Delta \mathbf{L}$ to angular momentum $\mathbf{L}$ of a spacecraft. Entire motion trajectory will consist of alternating active and passive sections and include accelerating and decelerating sections, sections of free motion ($\mathbf{M}_c = 0$), and short-term sections of trajectory correction. Problem of control consists of providing initial conditions for such uncontrolled sections, where predicted motion travels through the required position Λ_{pr} . At first correction, the predicted calculated spacecraft position Λ^* is taken: $\Lambda_{\text{pr}} = \Lambda_f \circ \tilde{\Lambda}(t_1) \circ \Lambda^*$, and quaternion of turn for computation of first correction impulse Λ_c is $\Lambda_c = \tilde{\Lambda}(t_1) \circ \Lambda_f \circ \tilde{\Lambda}(t_1) \circ \Lambda^*$. At all other corrections of spacecraft motion, $\Lambda_{\text{pr}} = \Lambda_f$ and $\Lambda_c = \tilde{\Lambda}_j \circ \Lambda_f$.

It was assumed in prognostic model that spacecraft is dynamically symmetric with respect to longitudinal axis and that disturbing moments are negligibly small. Specificity of this model is prediction of “free” motion of a spacecraft, in class of regular precession of rigid body. If we, taking this into account, solve kinematic problem of attitude with aim of transferring a spacecraft from position Λ_0 to position Λ_{pr} , we get the calculated value of vector of angular momentum $\mathbf{L}^*$. Velocities ω_{10} , ω_{20} , ω_{30} required for next section of free motion are determined from condition of fuel consumption minimum for the following control of spacecraft turn. It is evident that, in neighborhood of the calculated vector of angular velocity ω^* , we can consider its direction as fixed. In this case, fuel consumption G is function of only magnitude of angular velocity vector, which should be optimized. Sections of acceleration and deceleration coincide with predicted trajectories (because disturbance moment $\mathbf{M}_d$ is much less than control moment $\mathbf{M}_c$), and their duration is determined by time of a turn, value of control moment that could be achieved, and

quaternion of a turn. Duration of free motion sections is determined from condition of minimization of the functional G . Thus, control of spacecraft turn is reduced to successive realization of the following operations:

(1) Calculation of turn quaternion $\Lambda_t = \tilde{\Lambda}_{\text{in}} \circ \Lambda_f$ and determination of initial angular velocities for passive section ω_{10} , ω_{20} , ω_{30} ; prediction of spacecraft angular position Λ^* to instant of first correction $\text{sqa}(\tilde{\Lambda}_{\text{in}} \circ \Lambda^*) = \text{sqa}(\Lambda^* \circ \tilde{\Lambda}_f)$; determination of the required angular momentum $\mathbf{L}^*$ and control moment $\mathbf{M}$; we set $\Lambda_0 = \Lambda_{\text{in}}$;

(2) Acceleration of a spacecraft to the required angular momentum, and magnitude of control moment is maximal; accelerating torque is $\mathbf{M}_c = \tilde{\Lambda} \circ \mathbf{M}_{\text{ac}} \circ \Lambda$; and $\mathbf{M}_{\text{ac}}$ is maximal accelerating torque in inertial coordinate system, $\mathbf{M}_c \cdot \mathbf{L} > 0$;

(3) Free motion of a spacecraft ($\mathbf{M}_c = 0$) up to instant when $\text{sqa}(\tilde{\Lambda}_0 \circ \Lambda(t_j)) = \text{sqa}(\Lambda(t_j) \circ \tilde{\Lambda}_f)$, i.e., up to half of turn angle;

(4) At instant of time t_j , determination of new turn quaternion $\Lambda_t = \tilde{\Lambda}(t_j) \circ \Lambda_{\text{pr}}$ (moreover $\Lambda_{\text{pr}}^{(1)} = \Lambda_f$;

$\tilde{\Lambda}(t_1) \circ \Lambda^*$, and $\Lambda_{\text{pr}}^{(j)} = \Lambda_f$ if $j > 1$) and determination of initial angular velocities for new section (new hitting trajectory) ω_{1j} , ω_{2j} , ω_{3j} ; determination of the required impulse of angular momentum $\Delta \mathbf{L}$ equal $\Delta L_1 = J_1(\omega_{1j} - \omega_{11})$, $\Delta L_2 = J_2(\omega_{2j} - \omega_{21})$, $\Delta L_3 = J_3(\omega_{3j} - \omega_{31})$ and transfer of it to body of a spacecraft; here control moment $\mathbf{M}_c = \Delta \mathbf{L} / \Delta t$ (Δt is such that condition (4) is satisfied; we assume that $t_{\text{in}} = t_j$ and repeat points (3) and (4) up to instant of time $t = t_f - \tau$;

(5) Slowing down the angular velocities of a spacecraft with maximum control moment $\mathbf{M}_c \cdot \mathbf{L} < 0$, $\mathbf{M}_c = \tilde{\Lambda} \circ \mathbf{M}_{\text{br}} \circ \Lambda$

(i.e., control moment is directed exactly against the vector of angular momentum).

Scheme of iterative control that is proposed here allows one to take into account random factors on previous stages of spacecraft motion, thus decreasing the fuel consumption needed for control in further corrections. This is achieved due to exploiting the information about random and stochastic factors concerning spacecraft motion before first correction of angular momentum. Quaternion $\Delta \Lambda = \tilde{\Lambda}^* \circ \Lambda_1$ contains information concerning stochastic factors, random acts and perturbations, and factors that were not taken into account in control law. If we introduce it in sight parameters while forming the first correcting impulse, trajectory of free motion will be passed with lesser deviation from the required final position Λ_f (and with lesser expenditures needed for its compensation). Subsequent corrections are realized by guidance at final position (“reaiming” is done every time, from

current position Λ to final one, Λ_f). The obtained control law for spacecraft turn is sufficiently close to optimal one and allows one to decrease significantly the error in reducing the axes related with the spacecraft to a fixed final position Λ_f , in relation to other algorithms for realizing the method of guidance by a required velocity together with method of free trajectories. If conditions of turn Λ_{in} , Λ_f , and time T are such that times of acceleration and braking are very small (in comparison with total time of turn T) and we may neglect them, then one can consider as impulsive processes both imparting necessary angular momentum L_m to spacecraft and reducing available angular momentum down to zero, and almost during all turn (between acceleration and braking) $|\mathbf{L}(t)| = \text{const} = L_m$. These control algorithms can be easily realized by existing onboard means.

If durations of acceleration and braking are much smaller than duration of turn T , then the torque $\mathbf{M}$ is directed strictly against angular momentum $\mathbf{L}$ at spacecraft braking, and instant when braking begins can be predicted with high accuracy. Duration of rotation damping is $\tau = |\mathbf{L}|/m_0$ [11]. Instant of beginning of braking segment is determined by the condition [26]:

$$4\arcsin \frac{K\sqrt{\delta_2^2 + \delta_3^2}}{\sqrt{(J_2\omega_2)^2 + (J_3\omega_3)^2}} = \frac{K^2\sqrt{\omega_2^2 + \omega_3^2}}{m_0\sqrt{(J_2\omega_2)^2 + (J_3\omega_3)^2}}$$

where $\delta_1, \delta_2, \delta_3$ are components of vector part of mismatch quaternion $\tilde{\Lambda}(t) \circ \Lambda_f$; $K = |J\omega|$ is magnitude of spacecraft's angular momentum. At braking segment, cancellation of angular momentum is carried out according to linear law: $|\mathbf{L}(t)| = L_m - m_0(t - t_{br})$, where t_{br} is instant of beginning of braking.

Thus, we solved the problem of control for a programmed turn optimal with respect to fuel consumption on basis of algorithm with prognostic model. We investigated case when spacecraft's inertial characteristics are not exactly known in advance. Optimal solution to this problem is obtained in class of controls realized by method of free trajectories. Numerical realization of algorithm for coincident synthesis of optimal control in process of spacecraft turn is affected. Effective methods of control of terminal reorientation of a spacecraft is presented, one of which additionally has adaptive qualities - it is invariant with respect to external perturbations and substantially insensitive with respect to parametric errors. Indices of quality (economy and accuracy) of obtained laws are sufficiently high. Relatively low level of fuel consumption for a turn is achieved due to the transfer from permanent control of spacecraft attitude to formation of control moments only at certain definite instants of time. High accuracy of attitude is achieved by correcting angular momentum of a

spacecraft by varying its angular momentum up to its calculated value during reorientation process, at stage of free rotation, at discrete instants of time. Determination of time instant t_{br} according to actual (the measured values) kinematic parameters of motion (angular mismatch and angular velocity) improve accuracy of bringing the spacecraft into the required state $\Lambda = \Lambda_f, \omega = 0$.

6. Example of Numerical Solving the Control Problem and Results of Mathematical Modeling

Let us provide numerical solution of spacecraft's optimal control problem with respect to a programmed rotation. We consider maneuver from initial attitude Λ_{in} , when body axes coincide with axes of the supporting basis $\mathbf{I}$, into the given final position Λ_f with the following elements:

$$\lambda_0 = 0, \lambda_1 = 0.8, \lambda_2 = 0.6, \lambda_3 = 0$$

Spacecraft rotates from state of rest to state of rest, therefore initial and final angular velocities are zero: $\omega(0) = \omega(T) = 0$. We assume that maximum possible magnitude of the controlling moment m_0 and spacecraft's principal central inertia moments have values:

$$m_0 = 75 \text{ N m}, J_1 = 63559.2 \text{ kg m}^2, J_2 = 192218.5 \text{ kg m}^2, \text{ and } J_3 = 176808.9 \text{ kg m}^2$$

Also, we assume that duration of reorientation maneuver should be 360 seconds, approximately. As result of solving kinematic reorientation problem on transition from position $\Lambda(0) = \Lambda_{in}$ into position $\Lambda(T) = \Lambda_f$ (optimal rotation problem in impulse statement), we obtained value of ort of spacecraft's angular momentum for end of acceleration segment $\mathbf{p}_0 = \{0.600828; 0.451445; 0.659699\}$, if assume that spacecraft is dynamically symmetric body.

Optimal motion of a spacecraft consists of segments on which control moment maximum in magnitude acts (segments of acceleration and braking), of segments of free rotation, and of several corrections of angular motion within stage between acceleration and braking. On segment of maximal control moment, angular momentum vector $\mathbf{L}$ has permanent direction in inertial space, but it is variable in magnitude (increase up to preset value on acceleration segment, and decrease to zero on braking segment), while moment $\mathbf{M}$ is immovable with respect to reference basis $\mathbf{I}$ (the vectors $\mathbf{M}$ and $\mathbf{L}$ are parallel). During spacecraft rotation with maximum angular momentum modulus, parameters of motion are supported maximum nearby to the programmed values by impulses of the control-

ling moment. In this case, angular momentum vector $\mathbf{L}$ has approximately constant magnitude L_m between acceleration and braking. During correction impulses direction of angular mom

entum L varies from preset position to direction required for a hitting to final Λ_f . The calculated duration of acceleration (braking) is $\tau = 12$ s. Maximal magnitude of angular momentum (the programmed level) is $L_m = 900$ N m s.

Results of mathematical modeling of rotation process under optimal control in accordance with the method of guidance by a required velocity are demonstrated in Figures 1-4. Turn's duration was $T=360.24$ s. It means that perturbations (including asymmetry of the spacecraft) complicate rotation into required position. Visual illustration of rotation dynamics is given in Figure 1, where we present graphs of the changing angular velocities $\omega_1(t)$, $\omega_2(t)$, and $\omega_3(t)$ in time. In Figure 2 we present graphs of the changing components of quaternion $\Lambda(t)$, determining spacecraft's current attitude during rotation: $\lambda_0(t)$, $\lambda_1(t)$, $\lambda_2(t)$, and $\lambda_3(t)$. Figure 3 shows dynamics of the changing the components $p_1(t)$, $p_2(t)$, and $p_3(t)$ of ort $\mathbf{p}$ of angular momentum. The following rule is observed for functions $\omega_1(t)$ and $p_1(t)$: these functions are sign functions of time for any combinations of boundary values Λ_{in} and Λ_f . From Figure 1 and Figure 3, we see that number of motion corrections is five. Figure 4 shows character of changing the controlling moment, where we see all phases of controllable turn: acceleration of a spacecraft up to the programmed angular momentum, free motion, braking of spacecraft rotation, and short-term impulses of correction. Corrections of spatial motion are formed by the law that is described in section 4. In Figure 5, we see variations of angles ψ_f and ψ_0 ; ψ_f is smooth monotonically decreasing function of time, ψ_0 is piecewise continuous function of time, which is monotonically increasing function of time within intervals of continuity (between corrections). Instants of corrections are as follows:

$$t_1=179.2 \text{ s}, t_2=267.8 \text{ s}, t_3=312.72 \text{ s}, t_4=335.12 \text{ s}, t_5=346.4 \text{ s}$$

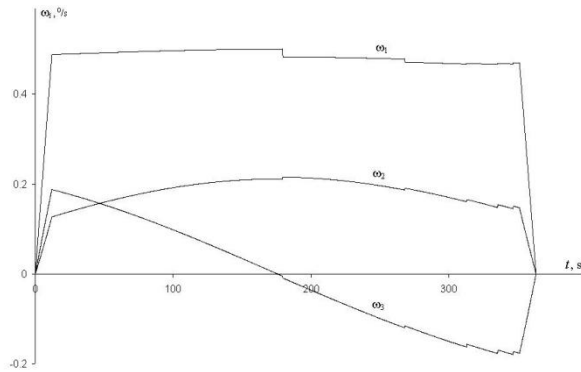


Figure 1. Optimal variation of angular rates during spatial reorientation

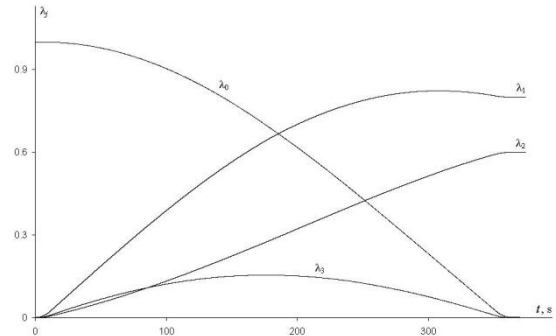


Figure 2. Variation of parameters of attitude during rotary maneuver

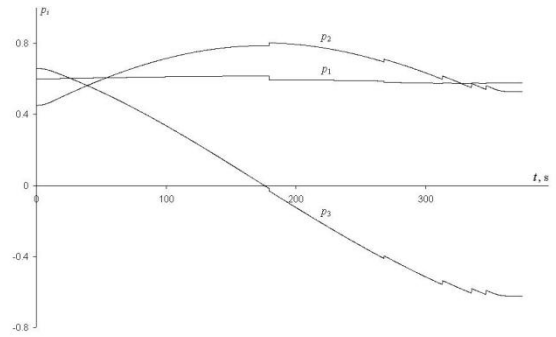


Figure 3. Elements of unit vector $\mathbf{p}$ as functions of time

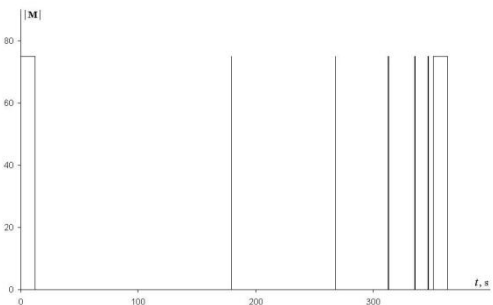


Figure 4. Changing the magnitude of control torque during optimal maneuver

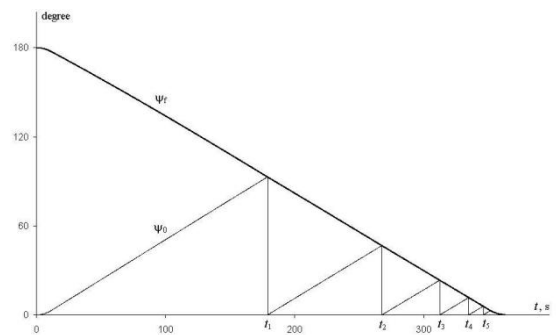


Figure 5. Angles ψ_f and ψ_0 and instants of corrections

Magnitudes of impulses of angular momentum for rotation correction are:

$$\Delta L_1 = 30 \text{ N m s}, \Delta L_2 = 24 \text{ N m s}, \Delta L_3 = 30 \text{ N m s}, \Delta L_4 = 36 \text{ N m s}, \text{ and } \Delta L_5 = 33 \text{ N m s}$$

Respectively, durations of correction impulses are as follows:

$$\Delta t_1 = 0.40 \text{ s}, \Delta t_2 = 0.32 \text{ s}, \Delta t_3 = 0.40 \text{ s}, \Delta t_4 = 0.48 \text{ s}, \Delta t_5 = 0.44 \text{ s}$$

Since Δt_n are small, the controlling moment for correction of motion can be calculated as follows:

$$M_{ci} = \frac{m_0(J_i \omega_{in} - J_i \omega_i)}{\sqrt{(J_1 \omega_{1n} - J_1 \omega_1)^2 + (J_2 \omega_{2n} - J_2 \omega_2)^2 + (J_3 \omega_{3n} - J_3 \omega_3)^2}}$$

where ω_{in} are the programmed values of angular velocities for next segment of free motion (for new hitting trajectory); t_n is instant of n -th correction; Δt_n is duration of n -th correction impulse.

Braking of a spacecraft requires some time and spacecraft rotates around angular momentum $\mathbf{L}$; the remaining angle φ_{rem} (for a turn around angular momentum from current position Λ into position Λ_f) and angle of spacecraft's rotation around vector $\mathbf{L}$ of angular momentum for time of braking φ_{br} have the following values (because modulus of angular momentum is changed linearly):

$$\varphi_{\text{rem}} = 2 \arcsin \frac{K \sqrt{\delta_2^2 + \delta_3^2}}{\sqrt{(J_2 \omega_2)^2 + (J_3 \omega_3)^2}},$$

$$\varphi_{\text{br}} = \frac{K^2 \sqrt{\omega_2^2 + \omega_3^2}}{2 m_0 \sqrt{(J_2 \omega_2)^2 + (J_3 \omega_3)^2}}$$

The remaining angle φ_{rem} is determined by relative orientation of current attitude Λ and the required final position Λ_f (for this purpose, we calculate quaternion of mismatch $\tilde{\Lambda}(t) \circ \Lambda_f$). Angle φ_{br} required for damping of angular momentum is determined by spacecraft's angular velocity. Variation of angles φ_{rem} and φ_{br} is shown in Figure 6. We begin a braking of a spacecraft when angles φ_{rem} and φ_{br} is identical (and difference $\Delta\varphi = \varphi_{\text{rem}} - \varphi_{\text{br}}$ is zero). If braking begin earlier, then spacecraft will not be moved to the required angular position Λ_f (spacecraft will stop before the required position Λ_f and not reach position Λ_f when rotation will end). If braking begin later, then spacecraft's angular velocity will be distinct from zero at the moment of achievement of angular orientation Λ_f . Deviation $\Delta\varphi$ is changed in accordance with Figure 7. At segment of braking, $\varphi_{\text{rem}} \approx \varphi_{\text{br}}$. It means that, at any instant within phase of braking, spacecraft can be turned through angle φ_{rem} during the remaining time of suppressing angular velocity up to zero; on the other hand, at any instant within phase of braking, spacecraft can be stopped to $\omega=0$

during a turn through the angle φ_{rem} (i.e. final position Λ_f will be achieved when $\omega=0$). Error of reorientation is $\sigma = 0.11$ degrees (accuracy depends on the acting disturbance moments - gravitational and aerodynamic moments - and due to inequality of transverse moments of inertia J_2 and J_3). Notice, angles ψ_f and $\psi_{\text{sub}} = |\boldsymbol{\omega}| |\mathbf{L}| / (2m_0)$ is changed in accordance with the Figure 8.

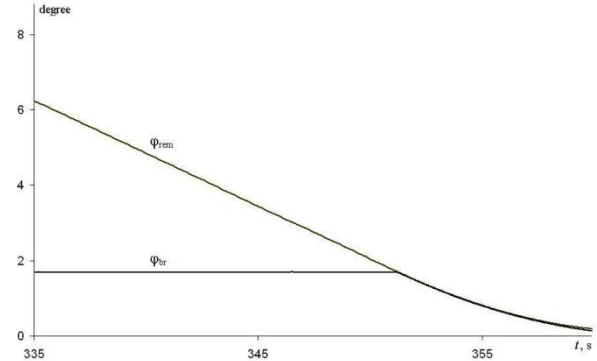


Figure 6. Variation of angles φ_{rem} and φ_{br} in ending of optimal turn

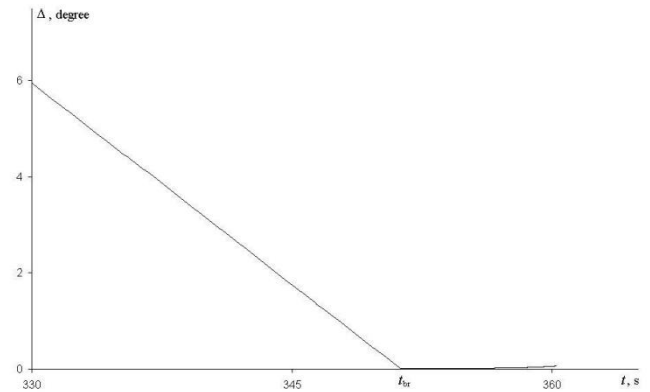


Figure 7. Character of deviation $\Delta\varphi$ before instant of beginning of braking and after it

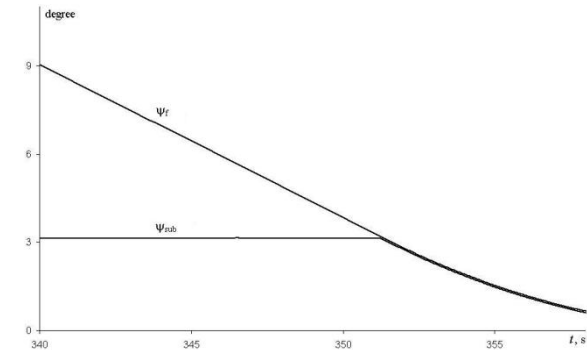


Figure 8. Angles ψ_f and ψ_{sub} before instant of beginning of braking and after it

Important characteristics of control are value of index (3) and accuracy of reorientation into the given final position. They were computed by mathematical simulation. Set of numerical experiments with modeling of spacecraft's rotation process was made. Ratings of control laws and estimates of efficiency of the designed control algorithms (precision of reorientation, and energy-saving efficiency) were calculated for each rotation maneuver in this series. Input data (original parameters of maneuvers - initial and final attitudes, spacecraft's inertial characteristics, and duration of rotation) were identical. Average modulus of angular rate during the rotary maneuvers is 0.5 deg/s. Estimates of fuel expenditure G and error of final attitude σ determined as result of the numerical simulations were: $G = 5.60$ kg with attitude accuracy $\sigma \approx 0.1^\circ$ correspond to iterative control by the method of guidance by a required velocity (without prediction of position at instant of first correction), and $G = 5.55$ kg with attitude accuracy $\sigma < 0.08^\circ$ correspond to mode of iterative guidance with prediction of position at instant of first correction. Also, for comparison, we cite values of same indicators of efficiency for same spacecraft which were specified after realizations of rotary maneuvers carried out according to mode of extensive rotation and rotation in form of regular precession (simultaneous rotation about longitudinal axis and about fixed transversal axis): $G_{ex} = 7.02$ kg and $G_{reg} = 6.13$ kg.

With help of mathematical modeling, for each method, we obtain statistical estimates of fuel consumption for one turn. They confirmed optimality of the designed control, which takes into account action of external moments of disturbance. Results of mathematical modeling demonstrate that the suggested methods of execution of the programmed turns improve accuracy of reorientation, given relatively large parametric undeterminacies and external perturbations. Moreover, "price" paid for absence of exact a priori information concerning dynamic characteristics of a spacecraft (some degradation of quality's index) is not too large. Let us note that traditional methods of optimal control not only require much more computational expenditure, but lead to greater fuel consumption with respect to control as well.

7. Conclusions

In this research, new control method of spacecraft attitude is presented. Example and results of mathematical simulation for spacecraft rotation under optimal control are given. The obtained results demonstrate that the designed control method of spacecraft's three-dimensional reorientation is feasible in practice.

Rotary maneuver is one of basic dynamic regimes of

motion control system. It was topical problem to design optimal algorithm of attitude control and to calculate numerical index of control efficiency. It seems impossible to solve the problem of slew maneuver of asymmetric spacecraft with minimal expense of fuel, taking into account disturbances (gravitational and aerodynamic torques). Algorithm of numerical constructing the control that satisfies all necessary requirements is suggested. It is limited in magnitude, ensures minimal consumption of fuel, and satisfies the given accuracy of attitude. Available mathematical model of spacecraft motion with respect to center of mass allowed us to use mode of guidance by a required velocity with prognostic model to form optimal control of attitude and to get its implementation. We know two-impulse control which rotate spacecraft along attitude trajectory consisting of three phases: fast imparting the required angular momentum to spacecraft's body with maximal control torque, free rotation without control torque, and quickest damping of rotation with maximal modulus of control torque. Each phase satisfies conditions of optimality: phase of free rotation fits condition of optimality, because there is no expenditure of fuel during it; phases of acceleration and braking also fit criterion of optimality, because minimal expenditure of fuel for acceleration and damping of rotation is determined only by the imparted angular momentum (which in turn is determined by inertial characteristics of a spacecraft, by initial and final conditions, and by duration of maneuver T). But error of reorientation can be unacceptable if external disturbing moments act long time (or angle of turn is large).

For improving the precision of attitude in the required position, one needs to control spacecraft's angular momentum in process of maneuver, using actual parameters of attitude. We present effective algorithms of controlling the reorientation, which are invariant under external disturbances and parametric discrepancy of model. Very effective mode is iteration control at which correction of rotation is carried out impulsively, at discrete instants of time. This mode ensures the required precision of attitude without refusal of control by method of free trajectories. Control commands are formed such that attitude actuators change angular momentum of a spacecraft not continuously, as at known methods, but impulsively at discrete instants of time. Since exact values of parameters of model of spacecraft rotation (e.g., moments of inertia) a priori are not known (they are known only roughly), to simplify onboard algorithms when angular rates are computed for points of beginning the uncontrolled phases, spacecraft is assumed dynamically symmetric body. The developed control laws are free from following typical simplifications: field of possible values of control

torques is not closed; the minimized index is quadratic function; there is constraint on angle of turn; spacecraft is dynamically symmetric about longitudinal axis; and finally, perturbation torques are neglected.

We give numerical estimates of fuel expense for implementation of rotary maneuver according to control modes which are presented above. Mathematical simulation allowed us to find values of fuel saving in performance of the described control laws for spacecraft's rotary maneuvers (in particular, for module of orbital station). Numerical modeling has shown that control which realize method of free trajectory with accounting for aerodynamic and gravitational models is sufficiently efficient and close to optimal solution. New control modes allow one to achieve considerable economy of fuel expense as compared to the known methods of control that is important for practice of spaceflight. In addition, these algorithms can be applied using modern onboard systems. The designed control methods of three-dimensional attitude allow expense of fuel for turn of existing spacecraft to be decreased by 20% - 30%. Estimates of fuel-saving efficiency of the presented modes of spacecraft attitude were calculated by statistical methods using numerical simulation in computer. Specific peculiarity of mathematical model of spacecraft rotation accepted in this paper for calculating the estimates of accuracy and fuel consumption is assumption about presence of significant aerodynamic and gravitational torques acting upon body of a spacecraft.

Notice, recent solutions^[19-22] are not applicable for general case of three-dimensional turn of arbitrary spacecraft; the work^[25] describes synthesis of terminal reorientation control only for spacecraft which moves along circular orbit. But method designed in present article is universal control, it does not depend on a ratio (proportion) of moments of inertia or final position of a spacecraft.

Acknowledgement

No financial exists.

Conflicts of Interest

No conflicts of interest exist.

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